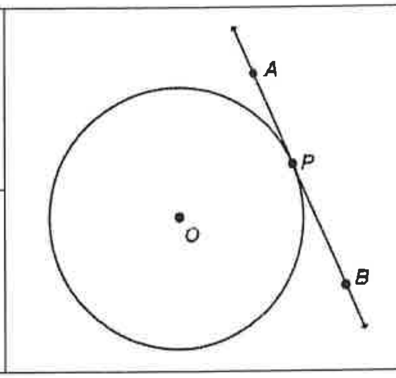
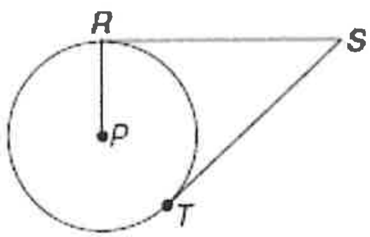
Tangent Lines of Circles

SWBAT solve for unknown variables using theorems about tangent lines of circles.

Tangent to a Circle Ex: (AB)	A line in the plane of the circle that intersects the circle in exactly one point. ***Ex:*** *Segment AB is a tangent to Circle O.*
Point of Tangency	The point where a circle and a tangent intersect. ***Ex:*** *Point P is a point of tangency on Circle O.*



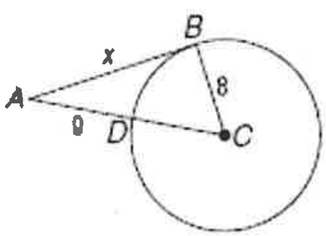
Tangent Theorem 1:	Converse Theorem 1:
If a line is tangent to a circle, then it is perpendicular to the radius draw to the point of tangency.	If a line is perpendicular to the radius of a circle at its endpoint on a circle, then the line is tangent to the circle.



Example: If RS is tangent, then PR _____ RS.

Example 1: Find the measure of x.

a)



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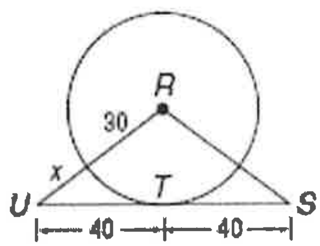
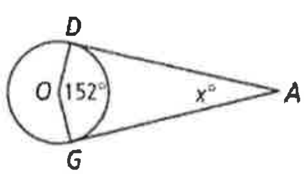


Example 2: Find x. *All segments that appear tangent are tangent to Circle O.*

a)



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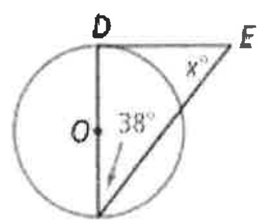
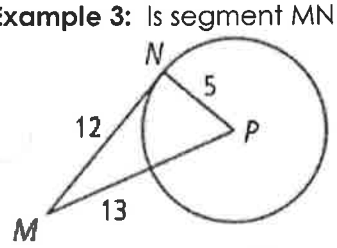
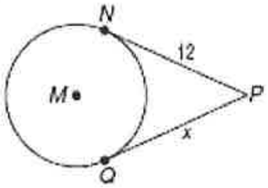


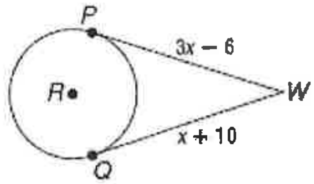
Example 3: Is segment MN tangent to Circle O at P? Explain.

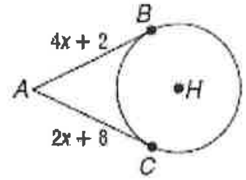


Tangent Theorem 2:	*If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.*

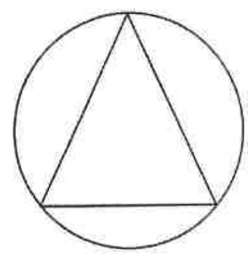
mple 4: Solve for x.



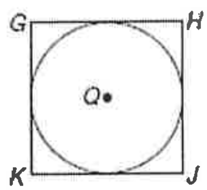


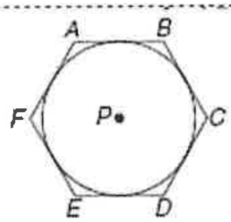


Circumscribed vs. Inscribed	
To ***circumscribe*** is when you draw a figure around another, touching it at points as possible.	To ***inscribe*** is to draw a figure within another so that the inner figure lies entirely within the boundary of the outer.
Ex: *The circle is circumscribed about the triangle.*	**Ex:** *The triangle is inscribed in the circle.*

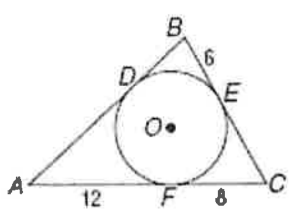


Tangent Theorem 3: (Circumscribed Polygons)	*When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.*



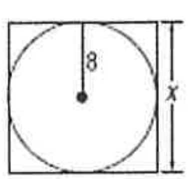


Example 5: Triangle ABC is circumscribed about ⊙O. Find the perimeter of triangle ABC.



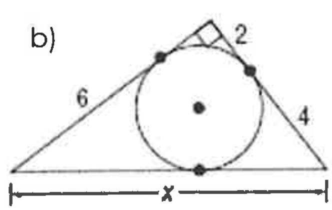
You Try! Find x. Assume that segments that appear to be tangent are tangent.

a)

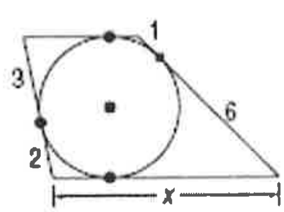


square

b)



c)



Math 3

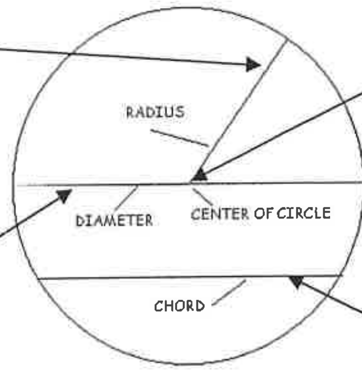
Chords & Arcs of Circles

Unit 6

SWBAT solve for unknown variables using theorems about chords and arcs of circles.

Any segment with ______________ that are the center and a point on the circle is a ____________.

A ___________ that passes through the center is a ___________ of a circle.

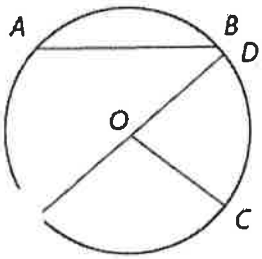


The given point is called the ___________.

This point names the circle.

Any segment with ________________ that are on a circle is called a ___________.

Example 1: Name the circle, a radius, a chord, and a diameter of the circle.



Circl
Radiu
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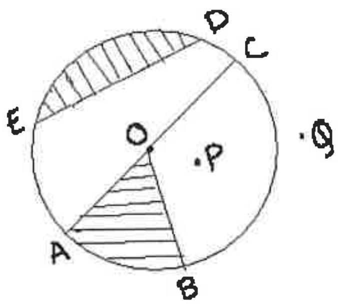
Diam



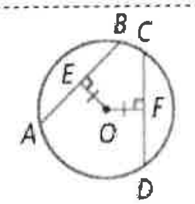
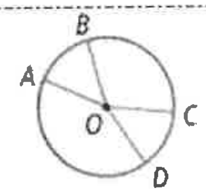
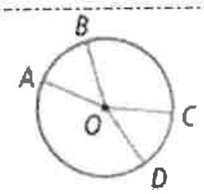
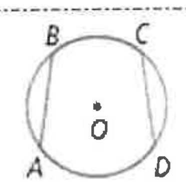
Circle: __________

Radius: __________

Chord: __________

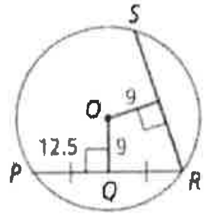
Diameter: __________

Since a _______________ is composed of two radii, then **d = 2r** and **r = d/2**

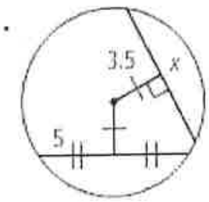
Theorem 1:	Converse Theorem 1:	
Within a circle or in congruent circles, chords equidistant from the center or centers are congruent. If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers). If $\overline{AB} \cong \overline{CD}$, then $OE = OF$.	
Theorem 2:	**Converse Theorem 2:**	
Within a circle or in congruent circles, congruent central angles have congruent arcs. If $\angle AOB \cong \angle COD$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.	Within a circle or in congruent circles, congruent arcs have congruent central angles. If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\angle AOB \cong \angle COD$.	
Theorem 3:	**Converse Theorem 3:**	
Within a circle or in congruent circles, congruent central angles have congruent chords. If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent chords have congruent central angles. If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.	
Theorem 4:	**Converse Theorem 4:**	
Within a circle or in congruent circles, congruent chords have congruent arcs. If $\overline{AB} \cong \overline{CD}$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.	Within a circle or in congruent circles, congruent arcs have congruent chords. If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\overline{AB} \cong \overline{CD}$.	

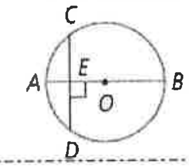
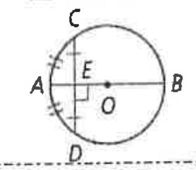
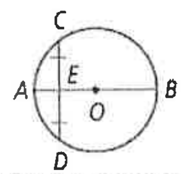
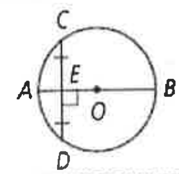
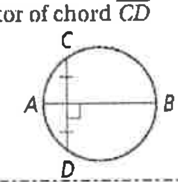
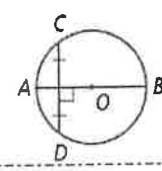
Example 2: The following chords are equidistant from the center of the circle.

a) What is the length of RS?

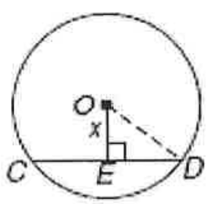


b) Solve for x.

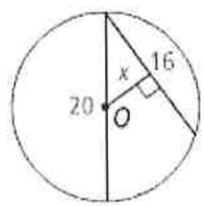


Theorem	If . . .	Then . . .
Theorem 5: In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.	$\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$ 	$\overline{CE} \cong \overline{ED}$ and $\overset{\frown}{CA} \cong \overset{\frown}{AD}$ 
Theorem 6: In a circle, if a diameter bisects a chord that is not a diameter, then it is perpendicular to the chord.	$\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$ 	$\overline{AB} \perp \overline{CD}$ 
Theorem 7: In a circle, the perpendicular bisector of a chord contains the center of the circle.	$\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$ 	$\overline{AB}$ contains the center of $\odot O$ 

Example 3: In $\odot O$, $\overline{CD} \perp \overline{OE}, OD = 15, and\ CD = 24$. Find x.

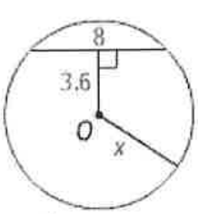


Example 4: Find the value of x to the nearest tenth.

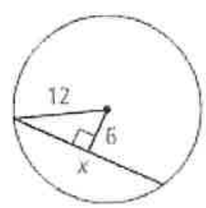


You Try! Find the value of x to the nearest tenth.

a)

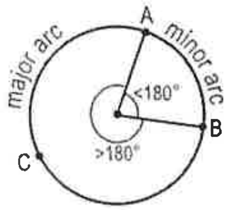


b)

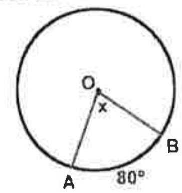


Inscribed Angles

SWBAT apply the rules and theorems of inscribed angles to solve for unknowns.



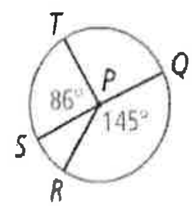
Major Arc:	Minor Arc:	Semicircle:
An arc of a circle measuring more than or equal to 180°	An arc of a circle measuring less than 180°	An arc of a circle measuring 180 °

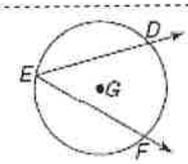
Central Angle:	A central angle is an angle formed by two intersecting radii such that its vertex is at the center of the circle.	
Central Angle Theorem:	*In a circle, or congruent circles, congruent central angles have congruent arcs.*	

Example 1: Identify the following in ⊙P at the right. For parts d-f, find the measure of each arc in ⊙P.

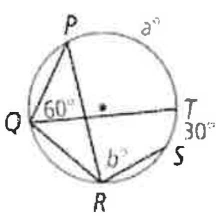
a) A semicircle b) A minor arc c) A major arc

d) $\widehat{ST}$ e) $\widehat{STQ}$ f) $\widehat{RT}$

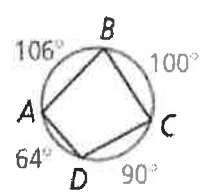


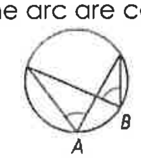
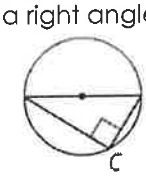
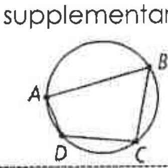
Inscribed Angle:	An inscribed angle is an angle with its vertex "on" the circle, formed by two intersecting chords.	
Inscribed Angle Theorem:	*The measure of an inscribed angle is half the measure of its intercepted arc.*	$m\angle DEF = \frac{1}{2} m\widehat{DF}$

Example 2: What are the values of a and b?



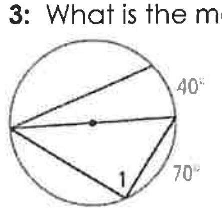
You Try! What are the m∠A, m∠B, m∠C, and m∠D?



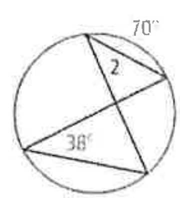
Corollary 1:	Corollary 2:	Corollary 3:
Two inscribed angles that intercept the same arc are congruent. 	An angle inscribed in a semicircle is a right angle. 	The opposite angles of a quadrilateral inscribed in a circle are supplementary. 

Example 3: What is the measure of each numbered angle?

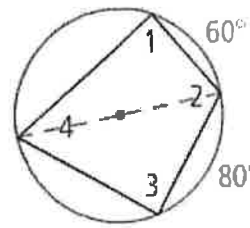
a)



b)



You Try! Find the measure of each numbered angle in the diagram to the right.

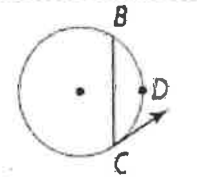


a) $m\measuredangle 1 =$

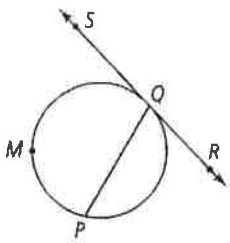
b) $m\measuredangle 2 =$

c) $m\measuredangle 3 =$

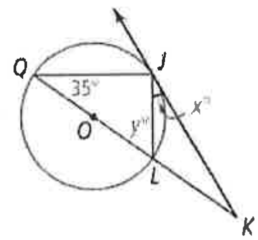
d) $m\measuredangle 4 =$

Tangent Chord Angle:	An angle formed by an intersecting tangent and chord has its vertex "on" the circle.	
Tangent Chord Angle Theorem:	*The tangent chord angle is half the measure of the intercepted arc.* **Tangent Chord Angle = ½ (Intercepted Arc)**	$m\angle C = \frac{1}{2} m\widehat{BDC}$

Example 4: In the diagram, $\overleftrightarrow{SR}$ is tangent to the circle at Q. If $m\widehat{PMQ} = 212$, what is the $m\angle PQR$?

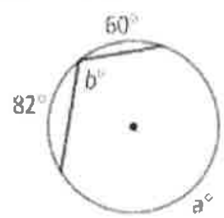


You Try! In the diagram, $\overleftrightarrow{KJ}$ is tangent to $\odot O$. What are the values of x and y?

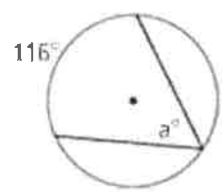


Practice: Find the value of each variable. For each circle, the dot represents the center.

1.



2.



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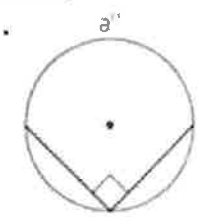

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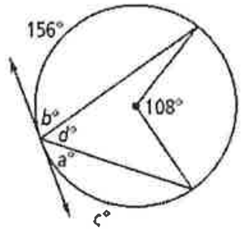

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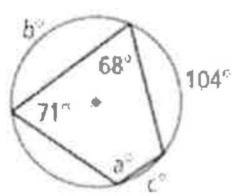
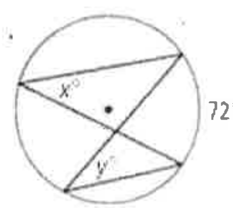


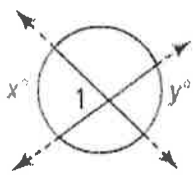
6.



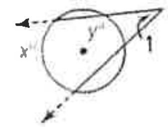
Angle Measures and Segment Lengths

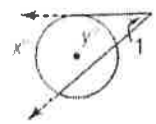
OBJ: Apply the rules and theorems of segments to solve for unknowns.

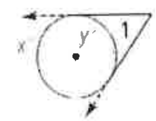
Theorem 1:	Theorem 2:
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.	The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(x + y)$



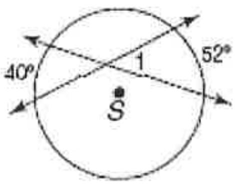




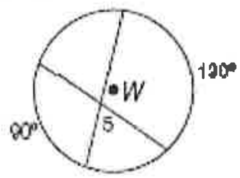
$m\angle 1 = \frac{1}{2}(x - y)$

Example 1: Find each measure.

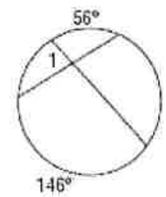
a) $m\angle 1$



b) $m\angle 5$

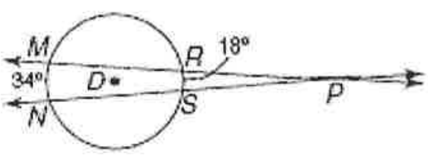


c) $m\angle 1$

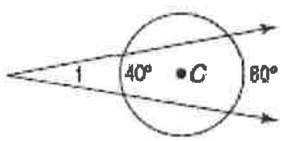


Example 2: Find the following angles.

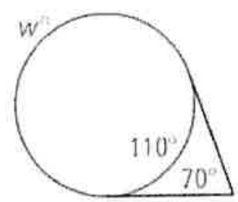
a) m∠MPN



b) $m\angle 1$

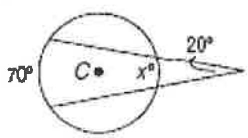


c)

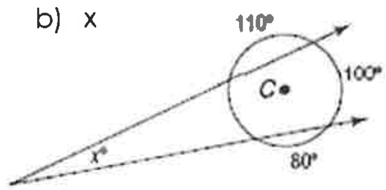


You Try! Find the following angles.

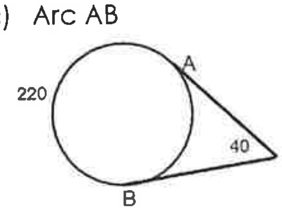
a) x



b) x

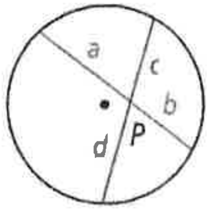


c) Arc AB

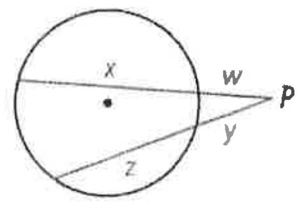


Theorem 3:

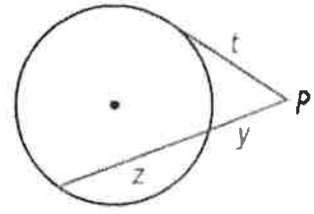
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and the circle.



$a \cdot b = c \cdot d$



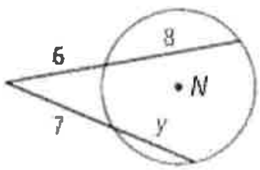
$(w + x)w = (y + z)y$



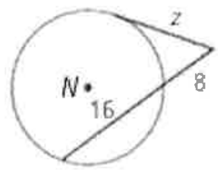
$(y + z)y = t^2$

Example 4: Find the value of the variable in ⊙ *O*.

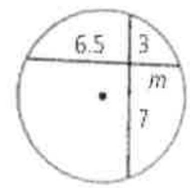
a)



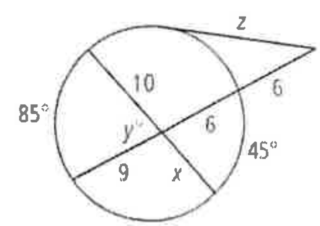
b)



c)



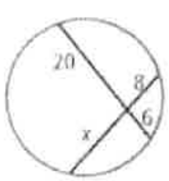
Try! What is the value of the variable to the nearest tenth?



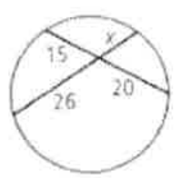
Algebra Find the value of each variable using the given chord, secant, and tangent lengths. If the answer is not a whole number, round to the nearest tenth.

See Problem 3.

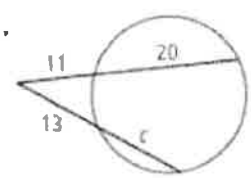
15.



16.



17.



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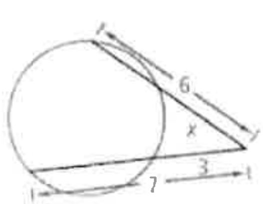

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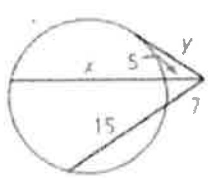
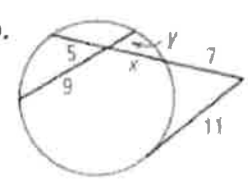


20.



Equations of Circles

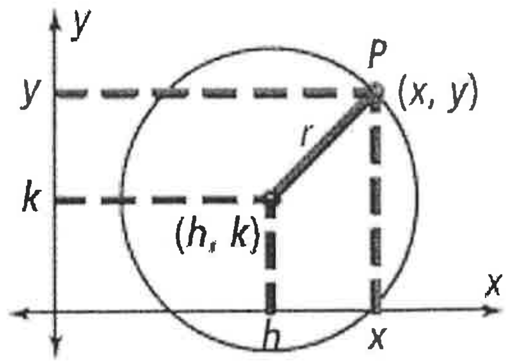
SWBAT graph circles on the coordinate plane and write the equations of circles in standard form.

Standard Form of Circles

Center:

Radius:

Point on the circle:



Example 1: Write the equation of a circle with the given information.

a) Center (0,0), Radius=10

h = k = r =

b) Center (2, 3), Diameter=12

h = k = r =

Example 2: Determine the center and radius of a circle the given equation.

a) $x^2 + y^2 = \frac{9}{4}$

b) $(x+3)^2 + (y-5)^2 = 81$

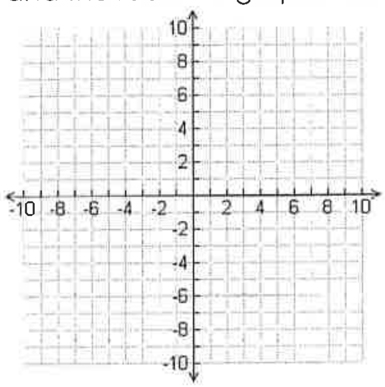
c) $(x+4)^2 + (y+6)^2 = 1$

Example 3: Use the center and the radius to graph each circle.

a) $(x+2)^2 + y^2 = 64$

Center:

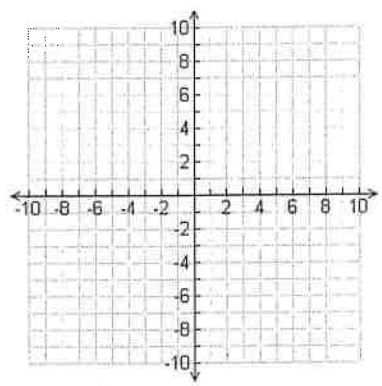
Radius:



b) $x^2 + (y+4)^2 = 36$

Center:

Radius:



Writing an Equation with a Pass-Thru Point
Step 1: Substitute the center (h, k) into the equation
Step 2: Substitute the "pass through point (x, y)" into the equation for x and y.
Step 3: Simplify and solve for r^2.
Step 4: Substitute r^2 back into the equation from Step 1.

Example 4: Write the equation of a circle with a given center (2, 5) that passes through the point (5 ,-1).

Writing an Equation with Two Points on the Circle	Midpoint Formula
Find the midpoint (radius) between the two endpoints, and then follow steps 1-4.	

Example 5: Write the equation of a circle with endpoints of diameter at (-6, 5) and (4, -3).

Writing the Equation of a Circle in Standard Form	
Step 1:	Group x's and group y's together.
Step 2:	Move any constants to the right side of the equation.
Step 3:	Use complete the square to make a perfect square trinomial for the x's and then again for the y's. **Remember, whatever you do to one side of the equation, you must do to the other!*
Step 4:	Simplify factors into standard form of a circle!

mple 5: Write the equation of a circle in standard form. Then, state the center and the radius.

a) $x^2 + y^2 + 4x - 8y + 16 = 0$

b) $x^2 + y^2 + 6x - 4y = 0$

c) $x^2 + y^2 - 6x - 2y + 4 = 0$

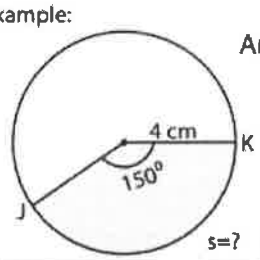
d) $x^2 + y^2 + 8x - 10y - 4 = 0$

Practice: Arc Length & Area of Sectors

Name: ____________________

Math 3

Example:



s=? Area=?

Arc length of a sector (s) = $\frac{\theta \times \pi \times r}{180^\circ}$

$= \frac{150^\circ \times 3.14 \times 4}{180^\circ}$

= 10.47 cm

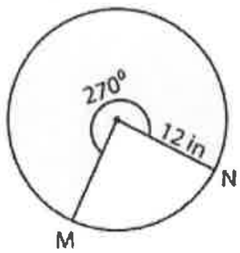
Area = $\frac{\theta \pi r^2}{360}$

$= \frac{150 \cdot 3.14 \cdot 4^2}{360}$

= 20.94 cm²

Find the length of the arc and area of the shaded region. Round the answer to two decimal places. (use π = 3.14)

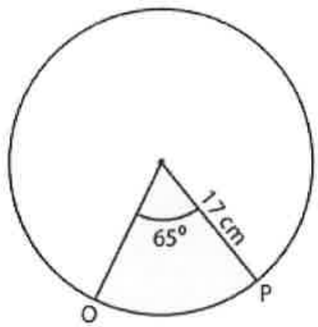
1)



Length of the arc MN = __________

Area of a sector = __________

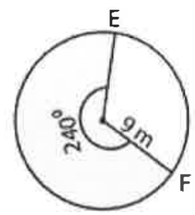
2)



Length of the arc OP = __________

Area of a sector = __________

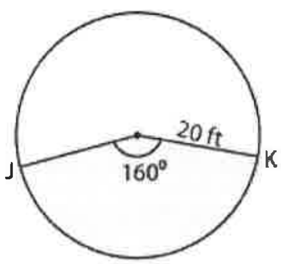
3)



Length of the arc EF = __________

Area of a sector = __________

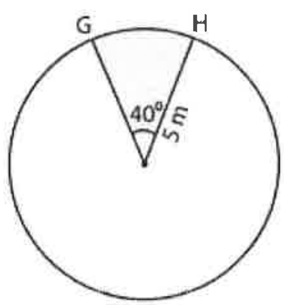
4)



Length of the arc JK = __________

Area of a sector = __________

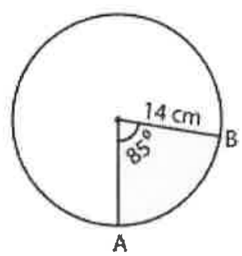
5)



Length of the arc GH = __________

Area of a sector = __________

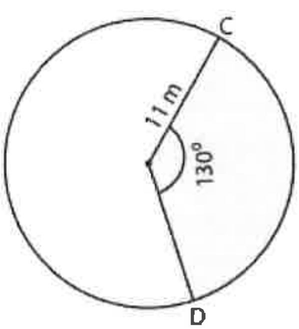
6)



Length of the arc AB = __________

Area of a sector = __________

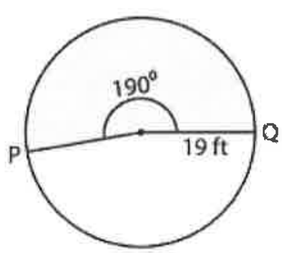
7)



Length of the arc CD = __________

Area of a sector = __________

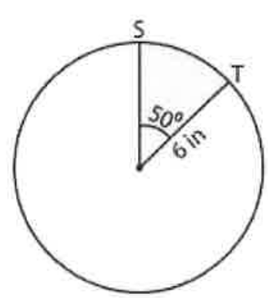
8)



Length of the arc PQ = __________

Area of a sector = __________

9)

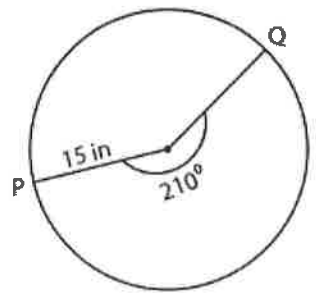


Length of the arc ST = __________

Area of a sector = __________

Find the missing one. Round the radius and central angle to the nearest whole number.
Round the arc length to two decimal places. (use $\pi = 3.14$)

1)

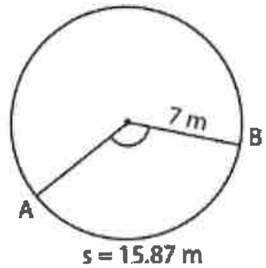


Radius = __________

Central angle = __________

Length of the arc PQ = __________

2)

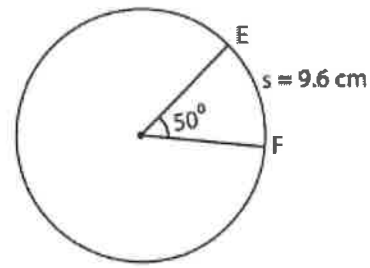


Radius = __________

Central angle = __________

Length of the arc AB = __________

3)



Radius = __________

Central angle = __________

Length of the arc EF = __________

Find the missing one. Round the radius and central angle to the nearest whole number.
Round the area to two decimal places. (use $\pi = 3.14$)

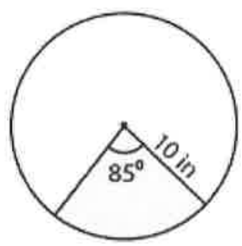
1)

Radius = __________

Central angle = __________

Area of a sector = __________

2)

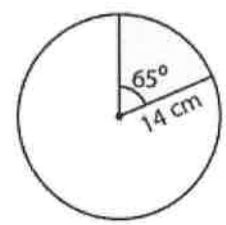


Radius = __________

Central angle = __________

Area of a sector = __________

3)



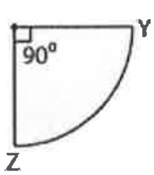
Radius = __________

Central angle = __________

Area of a sector = __________

Find the arc length for each sector. Round the answer to two decimal places. (use $\pi = 3.14$)

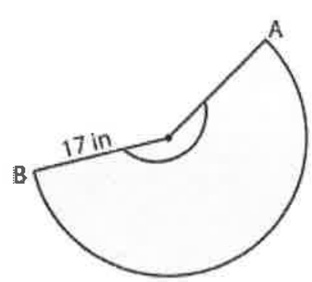
1)



Area = 153.86 cm^2

Length of the arc YZ = __________

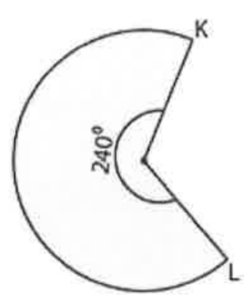
2)



Area = 529.35 in^2

Length of the arc AB = __________

3)



Area = 52.33 m^2

Length of the arc KL = __________