1.7 Solve Absolute Value Equations and Inequalities

Goal • Solve absolute value equations and inequalities.

Your Notes

VOCABULARY

Absolute value The absolute value of a number x, written $|x|$, is the distance the number is from 0 on a number line.

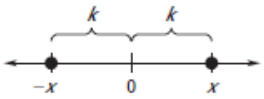
Extraneous solution An apparent solution that must be rejected because it does not satisfy the original equation.

INTERPRETING ABSOLUTE VALUE EQUATIONS

Equation $|x| = |x - 0| = k$

Meaning The distance between x and 0 is k.

Graph



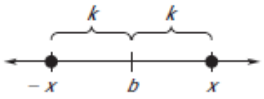
Solutions $x - 0 = -k$ or $x - 0 = k$

$x = -k$ or $x = k$

Equation $|x - b| = k$

Meaning The distance between x and b is k.

Graph



Solutions $x - b = -k$ or $x - b = k$

$x = b - k$ or $x = b + k$

$|6| = 6 \quad |-6| = 6$

SOLVING AN ABSOLUTE VALUE EQUATION

Use these steps to solve an absolute value equation $|ax + b| = c$ where $c > 0$.

Step 1 **Write** two equations: $ax + b =$ ___c___ or $ax + b =$ ___$-c$___.

Step 2 ___Solve___ each equation.

Step 3 **Check** each solution in the original ___absolute value___ equation. ✓

EXAMPLE 1 Solve a simple absolute value equation

Solve $|x - 5| = 7$. Graph the solution.

$x - 5 = 7 \quad x - 5 = -7$

$+5 \quad +5 \quad +5 \quad +5$

$x = 12 \quad x = -2$

EXAMPLE 2 Solve an absolute value equation

Solve $|5x - 10| = 45$.

EXAMPLE 3 Check for extraneous solutions

Always check your solutions in the original equation to make sure that they are not extraneous.

Solve $|2x + 12| = 4x$. Check for extraneous solutions.

$2x + 12 = 4x$

$-2x \quad -2x$

$12 = 2x$

$\frac{12}{2} = \frac{2x}{2}$

$6 = x$

$2x + 12 = -4x$

$-2x \quad -2x$

$12 = -6x$

$\frac{12}{-6} = \frac{-6x}{-6}$

$-2 = x$

GUIDED PRACTICE for Examples 1, 2 and 3

Solve the equation. Check for extraneous solutions.

4. $|3x - 2| = 13$

Solve the equation. Check for extraneous solutions.

5. $|2x + 5| = 3x$

Solve the equation. Check for extraneous solutions.

6. $|4x - 1| = 2x + 9$

$4x - 1 = 2x + 9$

$2x - 1 = 9$

$2x = 10$

$x = 5$

$4x - 1 = -(2x + 9)$

$4x - 1 = -2x - 9$

$6x - 1 = -9$

$+1 \quad +1$

$6x = -8$

$x = -\frac{4}{3}$

< = and > = or

ABSOLUTE VALUE INEQUALITIES

In the inequalities below, $c > 0$.

Inequality	Equivalent Form	Graph of Solution
$\lvert ax + b \rvert < c$	$-c < ax + b < c$	
$\lvert ax + b \rvert > c$	$ax + b < -c$ or $ax + b > c$	

In the inequalities shown at the right, ≤ can replace < and ≥ can replace > and the graphs would have solid dots.

$ax + b < c$

leave it

$ax + b > c$

change it

flip it

EXAMPLE 4 Solve an inequality of the form $|ax + b| > c$

Solve $|4x + 5| > 13$. Then graph the solution.

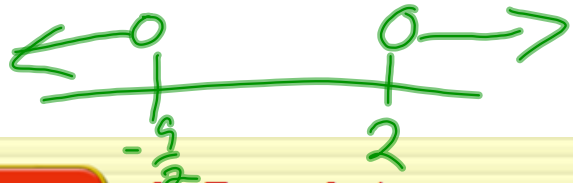
or

$4x+5>13$ $\quad$ $4x+5<-13$

$-5\ \ -5$ $\quad$ $-5\ \ -5$

$4x>8$ $\quad$ $4x<-18$

$x>2$ $\quad$ or $\quad x<-\frac{9}{2}$



GUIDED PRACTICE for Example 4

Solve the inequality. Then graph the solution.

7. $|x + 4| \geq 6$

Solve the inequality. Then graph the solution.

9. $|3x + 5| \geq 10$

GUIDED PRACTICE for Examples 5 and 6

Solve the inequality. Then graph the solution.

10. $|x + 2| < 6$

Solve the inequality. Then graph the solution.

12. $|7 - x| \leq 4$

EXAMPLE 5 Solve an inequality of the form $|ax + b| \leq c$

Baseball A professional baseball should weigh 5.125 ounces, with a *tolerance* of 0.125 ounce. Write and solve an absolute value inequality that describes the acceptable weights for a baseball.

$|X - \text{target value}| \leq \text{tolerance}$

$|X - 5.125| \leq .125$

$X - 5.125 \leq .125$ $\quad X - 5.125 \geq -.125$

$+5.125 \quad +5.125$

$X \leq 5.25$ $\quad X \geq 5.0$

EXAMPLE 6 Write a range as an absolute value inequality

Gymnastics

The thickness of the mats used in the rings, parallel bars, and vault events must be between 7.5 inches and 8.25 inches, inclusive. Write an absolute value inequality describing the acceptable mat thicknesses.

$7.25 \leq x \leq 8.25$