Unit 2 Polynomials

Introduction to Polynomials; Polynomial Graphs and Key Features

All Polynomials must have whole numbers as exponents!!

Example: $9x^{-1} + 12x^{\frac{1}{2}}$ is NOT a polynomial.

Polynomial Vocabulary Review

- **Expression:**
- **Equation:**
- **Terms:**
    - **Monomial, Binomial, Trinomial, Polynomial**
- **Degree:**
    - **Constant, Linear, Quadratic, Cubic, Quartic**

Example 1: Fill in the table below.

POLYNOMIAL	NUMBER OF TERMS	CLASSIFICATION BY TERMS	DEGREE	CLASSFICATION BY DEGREE	SKETCH THE GRAPH
$f(x) = 5$					
$g(x) = 4x - 3$					
$p(x) = -2x^5$					
$w(x) = x^4 - 4x + 2$					
$y = -4x^2 + x + 9$					
$h(x) = 4x^3 + x^2 - 9x + 2$					

You try! Write the following polynomial in standard form and classify by degree and number of terms.

$$P(x) = 3x^3 - 32x^2 + 48x + x^3$$

Degree: ____ Name Using Degree: __________ Name Using Number of Terms: __________

Sometimes, the polynomial is in factored form and looks like this:

$$y = x^2(-2x + 12)(x - 3$$

Degree: ____ Name Using Degree: __________ Name Using Number of Terms: __________

<u>Investigating End Behavior for Polynomial Functions</u>

Graph each function on your calculator. Use your graph to fill in the chart.

Graph	Is the degree even or odd?	Is the leading coefficient positive or negative?	Does the graph rise or fall on the left?	Does the graph rise or fall on the right?
1. $y = x^2$				
2. $y = x^4$				
3. $y = -x^2$				
4. $y = -x^4$				
5. $y = x^3$				
6. $y = x^5$				
7. $y = -x^3$				
8. $y = -x^5$				

End Behavior of a Polynomial Function				
	Leading coefficient is **Positive**		Leading coefficient is **Negative**	
	Left	Right	Left	Right
Degree is **odd**				
Degree is **even**				

You Try!!

Equation	Degree	Leading coefficient	End Behavior
1. $y = x^3 - 3x^2 - 3x + 9$			
2. $y = -2x^4 + 7x^2 - 6$			
3. $y = -3x^3 + x^2 + 10x - 5$			
4. $y = 8x^4 + 11x^3 + 5x^2$			
5. $y = -2x^4 - 5$			

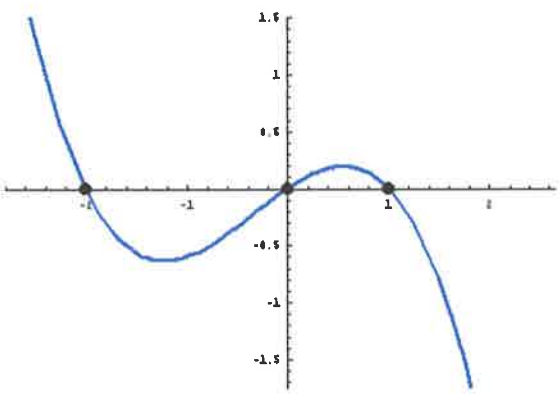
END BEHAVIOR----HAND BEHAVIOR

Degree	Leading Coefficient	Left-hand Behavior ($x \to -\infty$)	Right-hand Behavior ($x \to \infty$)	Picture
Even	Positive	$y \to \infty$	$y \to \infty$	
Even	Negative	$y \to -\infty$	$y \to -\infty$	
Odd	Positive	$y \to -\infty$	$y \to \infty$	
Odd	Negative	$y \to \infty$	$y \to -\infty$	

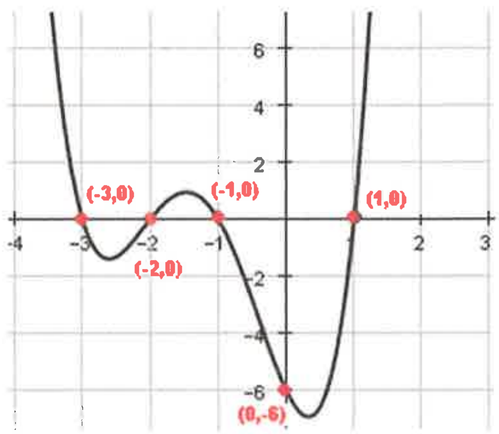
The number "k" is said to be a **zero** of a polynomial if f(k) = 0.

- "k" is often referred to as the **root** or **solution**
- If "k" is a real number, then f(k) = 0 means that the graph crosses the x-axis at that value. "k" can also be referred to as an **x-intercept**

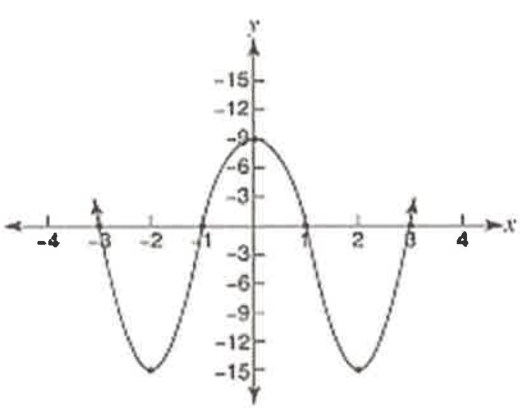
Check out the graphs below and identify any values that represent a zero/solution/root.



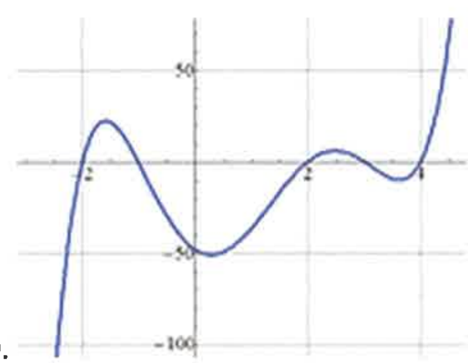
A.
factored equation: ______________________



B.
factored equation: ______________________



C.
factored equation: ______________________



D.
factored equation: ______________________

Multiplicity

Multiplicity

What is multiplicity? ____

____

A multiplicity that is an ODD number will result in the graph ____________ **or** ___________ **the x axis.**

A multiplicity that is EVEN will result in the graph ______________ **the x axis.**

Example 1. Consider $f(x) = x(x+3)(x-2)$

Example 2 $f(x) = -x(x+3)(x-2)$

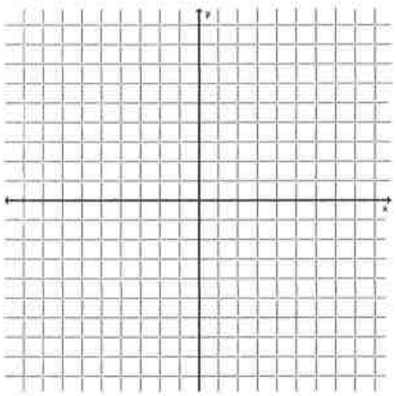
Example 3 $f(x) = x(x+3)^2(x-2)$

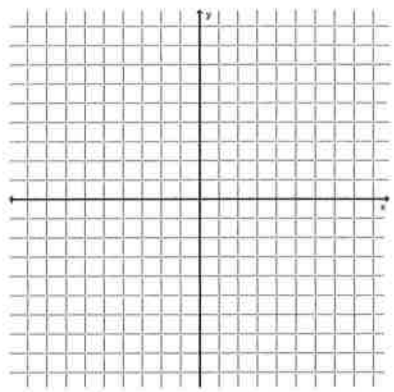
***Example 4:** In each of the following factored polynomials, what is the multiplicity of each zero?

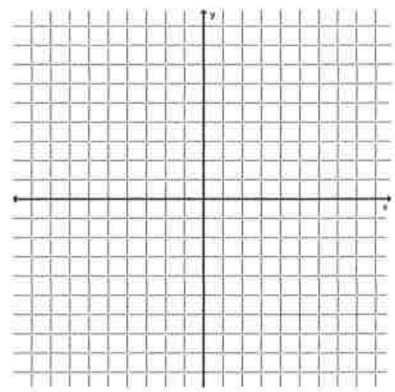
a) $y = (x-1)^2(x+2)^3(x-3)$ b) $y = -x(x-4)(x+5)^2$ c) $y = -(x-4)^2(x+6)$

*Using what you know about end behavior and multiplicity, sketch each of the polynomials from Example 4.

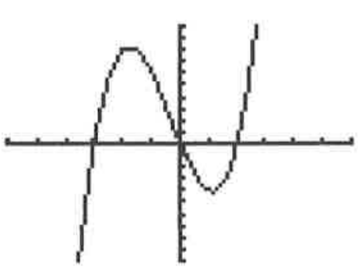
a) $y = (x-1)^2(x+2)^3(x-3)$ b $y = -x(x-4)(x+5)^2$ c) $y = -(x-4)^2(x+6)$

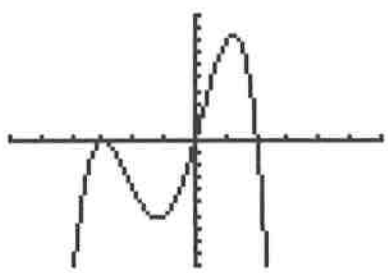


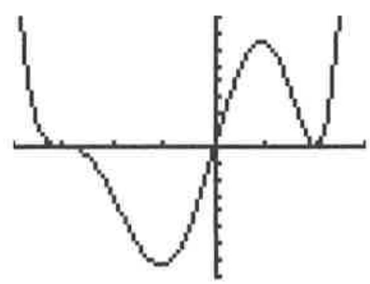




Use the graphs below to write polynomials that would fit the graphs, based on their zeros, end behavior, and multiplicity.







________________ ________________ ________________

Solve Polynomial Equations

Review Polynomial Factoring Techniques

Factor out the GCF $15x^4 - 20x^3 + 35x^2 =$ ______________________

Quadratic Trinomials $x^2 - 5x - 24 =$ ______________________

$6x^2 + 11x - 10 =$ ______________________

Perfect Square Trinomials $x^2 + 10x + 25$ ______________________

$x^2 - 10x + 25 =$ ______________________

Difference of Squares $4x^2 - 25y^2 =$ ______________________

Factor by Grouping $x^3 + 2x^2 - 3x - 6 =$ ______________________

Sum or Difference of Squares $x^3 + 8 =$ ______________________

$125y^3 - 64 =$ ______________________

Solve to find all real and imaginary solutions.

Ex. 1 $2x^3 - 5x^2 = 3x$	Ex 2 $3x^4 + 12x^2 = 6x^3$

Ex. 3 $x^4 - 4 = 0$	Ex. 4 $x^4 - 3x^2 = 4$
Ex. 5 $x^3 + 2x^2 + 5x + 10 = 0$	Ex. 6 $3x^4 + 10x^2 - 8 = 0$
Ex. 7 $x^3 - 64 = 0$	Ex. 8 $8x^3 - 1 = 0$

Dividing Polynomials, $f(x) = g(x)$ as a Solution

a) Using monomials

1. $\frac{(3a^2b+6a^3b^2+18ab)}{3ab}$

2. $\frac{12x^2y+3x}{3x}$

b) Long Division- Just like 4th grade!!!

$6\overline{)109}$

1. Ask yourself "what times the first term on the outside will give you the first term on the inside." Write this on top.

2. MULTIPLY this term by each term of the divisor.

3. Subtract this from the dividend.
   "Draw the line and change the sign

4. Repeat

   PUT PLACEHOLDERS IF YOUR POLYNOMIAL SKIPS A TERM

1. $x+3\overline{)x^2-5x-24}$

2. $\frac{9b^2+9b-10}{3b-2}$

3. $\frac{p^3-6}{p-1}$

4. $\frac{t^3-6t^2+1}{t+2}$

5. $(x^4 + 3x^3 + 5x^2 + 12x + 4) \div (x^2 + 3x + 1)$

c) <u>Synthetic Division (divisor MUST have leading coefficient of 1)</u>

1. **Coefficients in descending order**
2. **Write the ROOT from the given factor in the box (get it by setting the group = 0)**
3. **Bring down the first coefficient**
4. **Multiply, add, multiply, add…until finished**
5. **Write the answer, starting the polynomial as ONE degree lower than original**

****Placeholders are needed here as well****

. $(2x^3 - 7x^2 - 8x + 16) \div (x - 4)$

2. $(3x^3 - x^2 + 2x - 4) \div (x - 3)$

. $(z^4 - 20) \div (z + 2)$

4. $(2x^4 + 6x^3 + 5x - 6)(x + 2)$

. $(x^3 - x^2 + 2x - 7) \div (x - 1)$

The Factor Theorem, Remainder Theorem and Solutions of Two Polynomials

The Factor Theorem: The expression ________ is a factor of a polynomial if and only if the value ______ is a zero of the related polynomial function.

The Remainder Theorem: If you divide a polynomial by $x - a$, the remainder will be ______.

Example 1: Given that $P(x) = x^5 - 2x^3 - x^2 + 2$, use the Remainder Theorem to find if $x - 3$ is a factor of $P(x)$.

You try! Find the remainder if $P(x) = x^5 - 3x^4 - 28x^3 + 5x + 20$ is divided by $x - 4$.

Example 2: Is $(x + 2)$ a factor of $x^3 + 2x^2 - x - 2$?

You try! Is $x - 3$ a factor of $x^3 + 4x^2 + x - 6$? Is $x + 2$ a factor of the same poly?

Rational Zeros Theorem: How to find all ________________ zeros of a polynomial.

For a polynomial $P(x) = ax^3 + bx^2 + cx + d$, the potential zeros exist at ________________ where p will be theType equation here. factors of the constant term and q will be the factors of the leading coefficient.

Example 4: List the possible rational zeros of the function using the rational root theorem.

A. $f(x) = x^3 + 2x^2 - 11x + 12$

B. $f(x) = 4x^4 - x^3 - 3x^2 + 9x - 10$

Example 5: Find all real zeros of $f(x) = x^3 - 8x^2 + 11x + 20$

Example 6: Find all real zeros of the following:

A. $f(x) = x^3 - 4x^2 - 15x + 18$

B. $f(x) = x^3 - 8x^2 + 5x + 14$

You can also use the calculator!

Example 7: Graph the polynomial $x^4 - x^3 - 10x^2 - 8x$ to find a root. Then, divide to find all other factors.

Apply it! The polynomial $x^3 + 7x^2 - 38x - 240$ expresses the volume, in cubic inches, of a shadow box. What are the dimensions of the box? The length is greater than the height.

Polynomial Equations and Models

DIAMONDS The weight of an ideal round-cut diamond can be modeled by

$$w = 0.0071d^3 - 0.090d^2 + 0.48d$$

where w is the diamond's weight (in carats) and d is its diameter (in millimeters). According to the model, what is the weight of a diamond with a diameter of 15 millimeters?

CLOTHING The profit P (in millions of dollars) for a T-shirt manufacturer can be modeled by $P = -x^3 + 4x^2 + x$ where x is the number of T-shirts produced (in millions). Currently, the company produces 4 million T-shirts and makes a profit of $4,000,000. What lesser number of T-shirts could the company produce and still make the same profit?

MP3 PLAYERS The profit P (in millions of dollars) for a manufacturer of MP3 players can be modeled by $P = -4x^3 + 12x^2 + 16x$ where x is the number of MP3 players produced (in millions). Currently, the company produces 3 million MP3 players and makes a profit of $48,000,000. What lesser number of MP3 players could the company produce and still make the same profit?

SWIMMING POOL You are designing a rectangular swimming pool that is to be set into the ground. The width of the pool is 5 feet more than the depth, and the length is 35 feet more than the depth. The pool holds 2000 cubic feet of water. What are the dimensions of the pool?

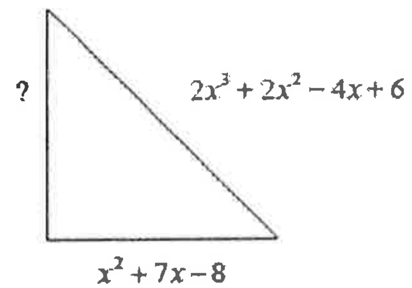
BUSINESS For the 12 years that a grocery store has been open, its annual revenue R (in millions of dollars) can be modeled by the function

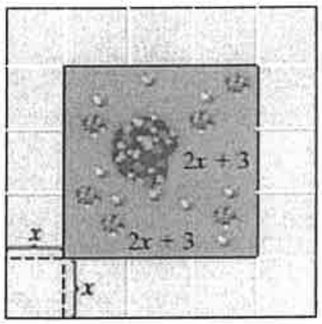
$$R = 0.0001(-t^4 + 12t^3 - 77t^2 + 600t + 13{,}650)$$

where t is the number of years since the store opened. In which year(s) was the revenue $1.5 million?

GIVEN THE PERIMETER, FIND THE MISSING SIDE.

$P = 2x^3 + 4x^2 + 6x + 3$

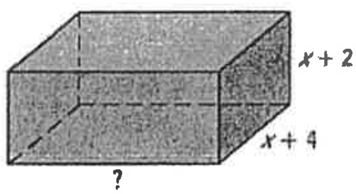




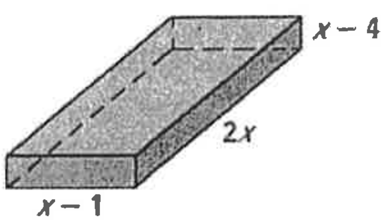
A) FIND THE AREA OF THE GARDEN.

B) FIND THE AREA OF THE WALKWAY AND THE GARDEN.

$V = 2x^3 + 17x^2 + 46x + 40$



Volume = 40



A metal worker wants to make an open box from a 12 in by 16 inch sheet of metal by cutting equal squares from each corner. Write a function for the volume of the box. Find the maximum volume of the box and the side length of the cut out squares that generate the volume.

A rectangular picture is 12 in. by 16 in. When each dimension is increased by the same amount, the area is increased by 60 in^2. If x represents the number of inches by which each dimension is increased, which equation could be used to find the value for x?